C Accuracy and Reliability

As stated in Section D,

$$C = \frac{\text{Real Liquid Flow Rate as Observed in the Flow Calibrating Facility}}{\text{Flow Rate Calculated from Equation 1 with } C = 1,\ Y = 1, \text{ Using the Observed Differential Pressure}}$$

The way C is used is shown in Equation 1 on page D1. It connects the idealized flow rate to the real one. Consequently, when you buy a flow meter, you are spending money only for the value and the accuracy of C (the discharge coefficient) and Y (the expansion factor). The reliability of this knowledge, therefore, is of great importance.

Due to its definition, C is a measure of the extent to which the real flow metering mechanism of a meter violates the ideal mechanism described by the "Ideal Flow For Liquids" portion of Equation 1. As a result, the value, R_D behavior, line size behavior, and beta ratio behavior of the C belong to the metering shape of a tube, controlling its possibility for accuracy and reliability when used for flow measurement.

The ultimate source of C knowledge is flow calibration with water in a calibration facility. Due to the fact that "invisible" C characteristics are tied to the "visible" metering shape of tubes, it is possible to transfer C knowledge found on one tube by flow calibration to other tubes which have the same metering shape when inspected on the bench.

Consequently, there are two ways in which Cs can be assigned to flow meters:

- By direct flow calibration of the device, in which case we refer to **flow calibrated C: C_F**
- By bench calibration. Here the meter itself is not exposed to flow, rather it is inspected on the "bench" to determine that its metering shape is within the prescribed manufacturing tolerances. A C derived from the flow calibration of other "meteringly same" devices that are built within the same manufacturing tolerances is then assigned to it. In such cases we speak about **bench calibrated C: C_B**

Due to its importance in flow measurement, we must explain "metering sameness" and describe the possibilities it creates for meter types possessing it and those it denies to meters lacking it. **A flow metering design principle secures metering sameness if meters built according to that principle have the same C characteristics, regardless of physical differences in line size and/or beta ratio.**

Metering sameness surmounts handicaps to flow measurement which stop meters that do not have it:

> **It makes large line size flow measurement with high accuracy and reliability possible.** C characteristics obtained on smaller line size meters, for which flow calibrations are available, can be accurately and reliably extrapolated for meters too large to be flow calibrated;
>
> **It permits the determination of flow pattern effects.** It allows for the extrapolation of knowledge obtained from flow calibration of smaller line size meters to any other line size meter;
>
> **It allows for the estimation of flow pattern effects for large line size installations** for which flow calibrating facilities are not available.

It was for these reasons that the HVT design principle was developed. It secures metering sameness for the entire product line.

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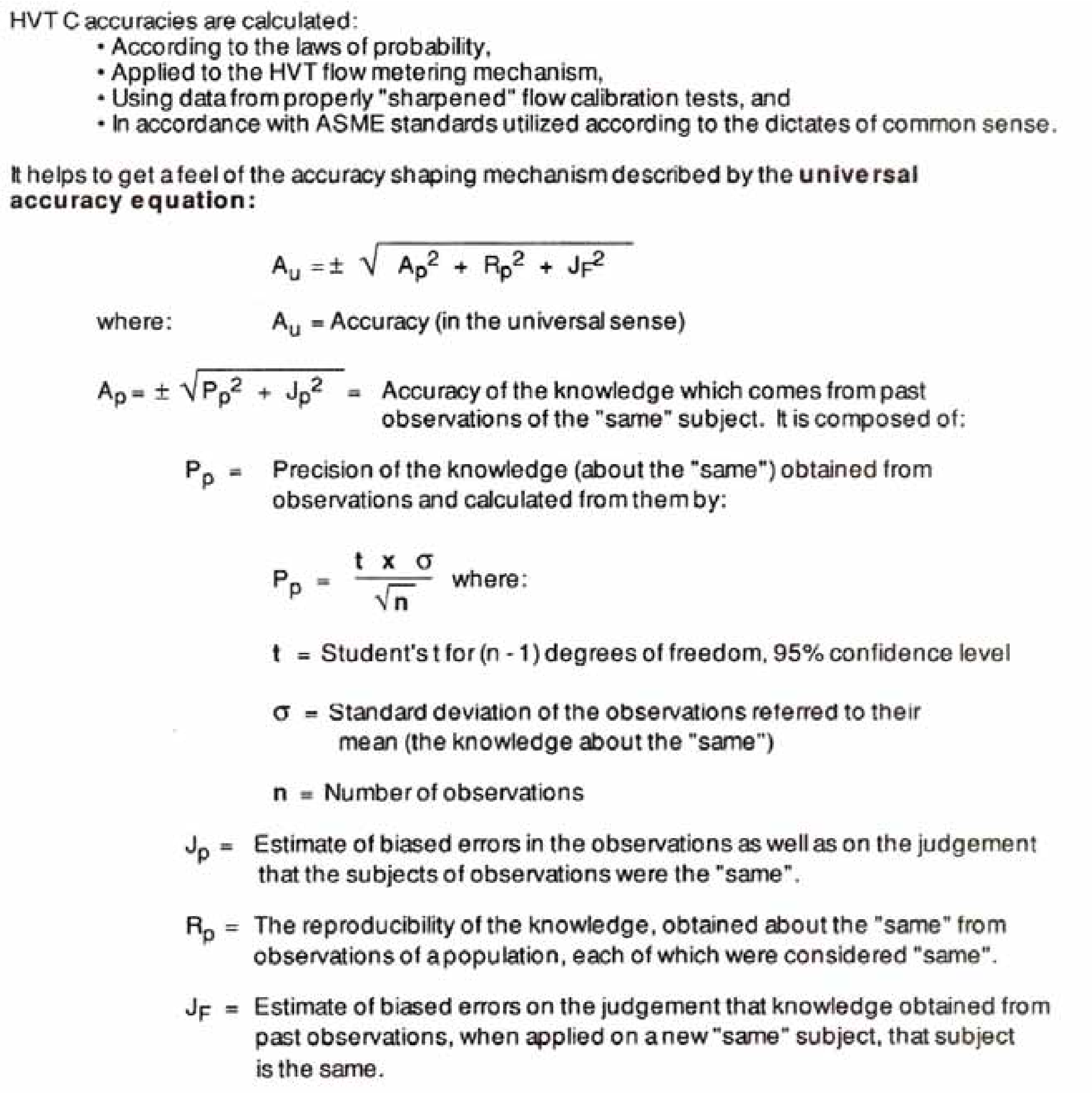
Determination of C Accuracy



HVT C accuracies are calculated:

- According to the laws of probability,
- Applied to the HVT flow metering mechanism,
- Using data from properly "sharpened" flow calibration tests, and
- In accordance with ASME standards utilized according to the dictates of common sense.

It helps to get a feel of the accuracy shaping mechanism described by the **universal accuracy equation:**

$$A_u = \pm\sqrt{A_p^2 + R_p^2 + J_F^2}$$

where: A_u = Accuracy (in the universal sense)

$A_p = \pm\sqrt{P_p^2 + J_p^2}$ = Accuracy of the knowledge which comes from past observations of the "same" subject. It is composed of:

P_p = Precision of the knowledge (about the "same") obtained from observations and calculated from them by:

$$P_p = \frac{t \times \sigma}{\sqrt{n}}$$ where:

t = Student's t for (n - 1) degrees of freedom, 95% confidence level

σ = Standard deviation of the observations referred to their mean (the knowledge about the "same")

n = Number of observations

J_p = Estimate of biased errors in the observations as well as on the judgement that the subjects of observations were the "same".

R_p = The reproducibility of the knowledge, obtained about the "same" from observations of a population, each of which were considered "same".

J_F = Estimate of biased errors on the judgement that knowledge obtained from past observations, when applied on a new "same" subject, that subject is the same.

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Determination of Flow Calibrated C Accuracy (A_F)

Based on the universal accuracy equation we can calculate specific HVT accuracies as follows:

$$A_F = \sqrt{P_f^2 + P_L^2 + R_L^2 + J_L^2} \quad \text{where:}$$

P_f = C precision caused by random errors in the flow calibration (twice sigma value)
P_L = C precision caused by random differences between the C effects of flow patterns considered "normal" (twice sigma value)
R_L = Reproducibility of the normal flow pattern effect at field installation of the meter (twice sigma value)
J_L = Bias error in the flow calibration not accounted for in above items (twice sigma equivalent)

The flow calibrated C accuracies for HVTs when installed in the field are tabulated below for 95% (twice standard deviation) confidence level:

A_F - Flow Calibrated C Accuracies for HVTs
With Static Inlet Tap (95% = 2 sigma) ±% of C

Beta	P_f	P_L	R_L	J_L	A_F
0.75	0.06	0.16	0.30	0.10	0.36
0.70	0.06	0.11	0.20	0.10	0.26
0.60	0.06	0.05	0.10	0.10	0.16
0.50	0.06	0.03	0.05	0.10	0.13
0.40	0.06	0.01	0.02	0.10	0.12
0.30	0.06	0.01	0.01	0.10	0.12

A_F - Flow Calibrated C Accuracies for HVTs
With Corner Inlet Tap (95% = 2 sigma) ±% of C

Beta	P_f	P_L	R_L	J_L	A_F
0.75	0.06	0.24	0.45	0.10	0.52
0.70	0.06	0.16	0.30	0.10	0.36
0.60	0.06	0.08	0.15	0.10	0.21
0.50	0.06	0.05	0.08	0.10	0.15
0.40	0.06	0.02	0.03	0.10	0.13
0.30	0.06	0.01	0.01	0.10	0.12

LIMITATIONS: Flow calibrations with the 0.10% bias uncertainty (J_L) can be obtained for:
Line sizes: 1" to 30"
Pipe Reynolds numbers (R_D):
Maximum: Determined by meter size, beta ratio, and water temperature
Minimum: Limited by the smallest differential pressure which can be read with sufficient accuracy.

NOTES: PFS develops a calibration arrangement and procedure which assures the greatest benefit from the flow calibration in order to achieve the best field installed accuracy for the meter. Commonly used upstream and downstream piping arrangements are available.

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Explanation of Bench Calibrated Accuracy

The bench calibrated C is determined through a chain of observations whose links are:

- Observing (inspecting) the metering shapes of the tubes to determine that they are "meteringly same" prior to the flow calibration, thereby providing the bench calibrated C for future meters;
- Observing the Cs of tubes through flow calibrations in calibrating facilities;
- Observing the metering shape of a new tube to determine that it is meteringly the same as those meters whose flow calibrated Cs determined the bench calibrated C. Consequently, the bench calibrated C can be assigned to the new tube.

The HVT design secures a single C, called C_t (the true C) regardless of line size and beta ratio. C_t can be determined only from the observed Cs (C_O) which, however, contain the errors of observations (E_O). Therefore, $C_O = C_t + E_O$.

Since C_t is the same for all tubes, and E_O is random by nature, the more tubes that are flow calibrated to establish the bench calibrated C (C_B), the better will be the C_B accuracy. This is due to the following mechanism:

$$\begin{aligned} C_{O1} &= C_t + E_{O1} \\ C_{O2} &= C_t + E_{O2} \\ &\vdots \\ C_{On} &= C_t + E_{On} \end{aligned}$$

Summing and taking the mean, we arrive at : $\frac{\sum^{n} C_O}{n} = \frac{nC_t}{n} + \frac{\sum^{n} E_O}{n} = \overline{C_O} = C_B$

Consequently, $C_B = C_t \pm \frac{\sum^{n} E_O}{n}$

The error term disappears if the number of observations is infinite, a state which can never be reached. It can be estimated, however, due to the algebra which describes its nature. Using our terminology, the

Precision of the Bench Calibrated C = $P = \frac{t \times \sigma}{\sqrt{n}}$ where:

t = Student's t for 95% confidence level and (n - 1) degrees of freedom

σ = Standard deviation = $\sqrt{\frac{\sum E_O^2}{n - 1}}$

E_O is estimated by: $E_O = \Delta C = \overline{C_O} - C_B$

n = Number of observations

P, as calculated above, determines the accuracy of C_B as established by past observations. To estimate the accuracy of a new tube to which C_B was assigned, the ability to reproduce C_B on the new tube must be estimated. This estimate of reproducibility (R) according to the laws of probability for 95% confidence level, is two sigma. Consequently, the bench calibrated C accuracy (A_B) is:

$$A_B = \sqrt{P^2 + R^2}$$

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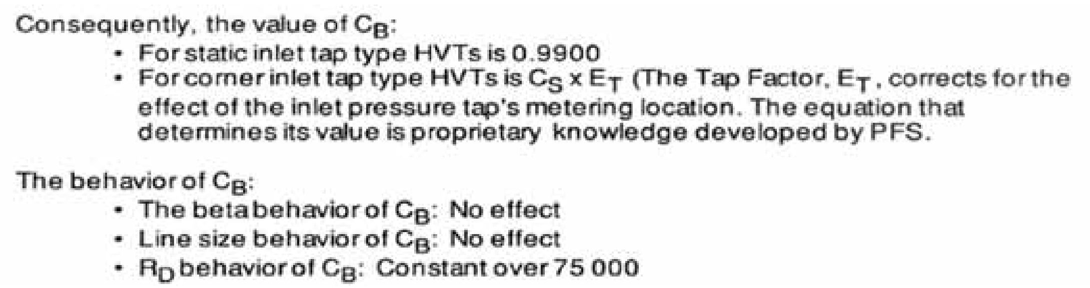
Explanation of Bench Calibrated Accuracy (continued



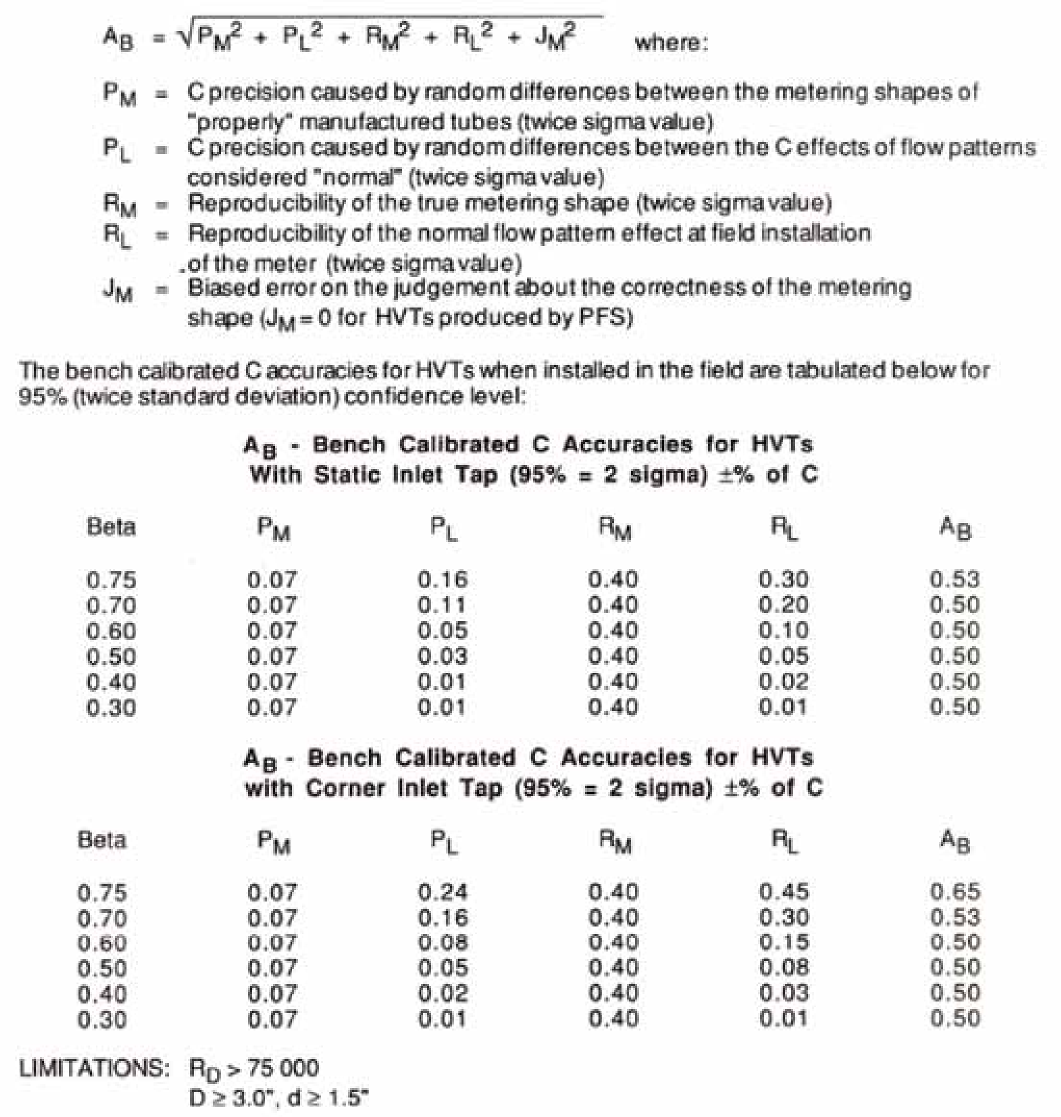
Consequently, the value of C_B:

- For static inlet tap type HVTs is 0.9900
- For corner inlet tap type HVTs is $C_S \times E_T$ (The Tap Factor, E_T, corrects for the effect of the inlet pressure tap's metering location. The equation that determines its value is proprietary knowledge developed by PFS.

The behavior of C_B:

- The beta behavior of C_B: No effect
- Line size behavior of C_B: No effect
- R_D behavior of C_B: Constant over 75 000

Determination of Bench Calibrated C Accuracy (A_B)



$$A_B = \sqrt{P_M^2 + P_L^2 + R_M^2 + R_L^2 + J_M^2}$$ where:

P_M = C precision caused by random differences between the metering shapes of "properly" manufactured tubes (twice sigma value)
P_L = C precision caused by random differences between the C effects of flow patterns considered "normal" (twice sigma value)
R_M = Reproducibility of the true metering shape (twice sigma value)
R_L = Reproducibility of the normal flow pattern effect at field installation of the meter (twice sigma value)
J_M = Biased error on the judgement about the correctness of the metering shape ($J_M = 0$ for HVTs produced by PFS)

The bench calibrated C accuracies for HVTs when installed in the field are tabulated below for 95% (twice standard deviation) confidence level:

A_B - Bench Calibrated C Accuracies for HVTs With Static Inlet Tap (95% = 2 sigma) ±% of C

Beta	P_M	P_L	R_M	R_L	A_B
0.75	0.07	0.16	0.40	0.30	0.53
0.70	0.07	0.11	0.40	0.20	0.50
0.60	0.07	0.05	0.40	0.10	0.50
0.50	0.07	0.03	0.40	0.05	0.50
0.40	0.07	0.01	0.40	0.02	0.50
0.30	0.07	0.01	0.40	0.01	0.50

A_B - Bench Calibrated C Accuracies for HVTs with Corner Inlet Tap (95% = 2 sigma) ±% of C

Beta	P_M	P_L	R_M	R_L	A_B
0.75	0.07	0.24	0.40	0.45	0.65
0.70	0.07	0.16	0.40	0.30	0.53
0.60	0.07	0.08	0.40	0.15	0.50
0.50	0.07	0.05	0.40	0.08	0.50
0.40	0.07	0.02	0.40	0.03	0.50
0.30	0.07	0.01	0.40	0.01	0.50

LIMITATIONS: $R_D > 75\ 000$
$D \geq 3.0"$, $d \geq 1.5"$

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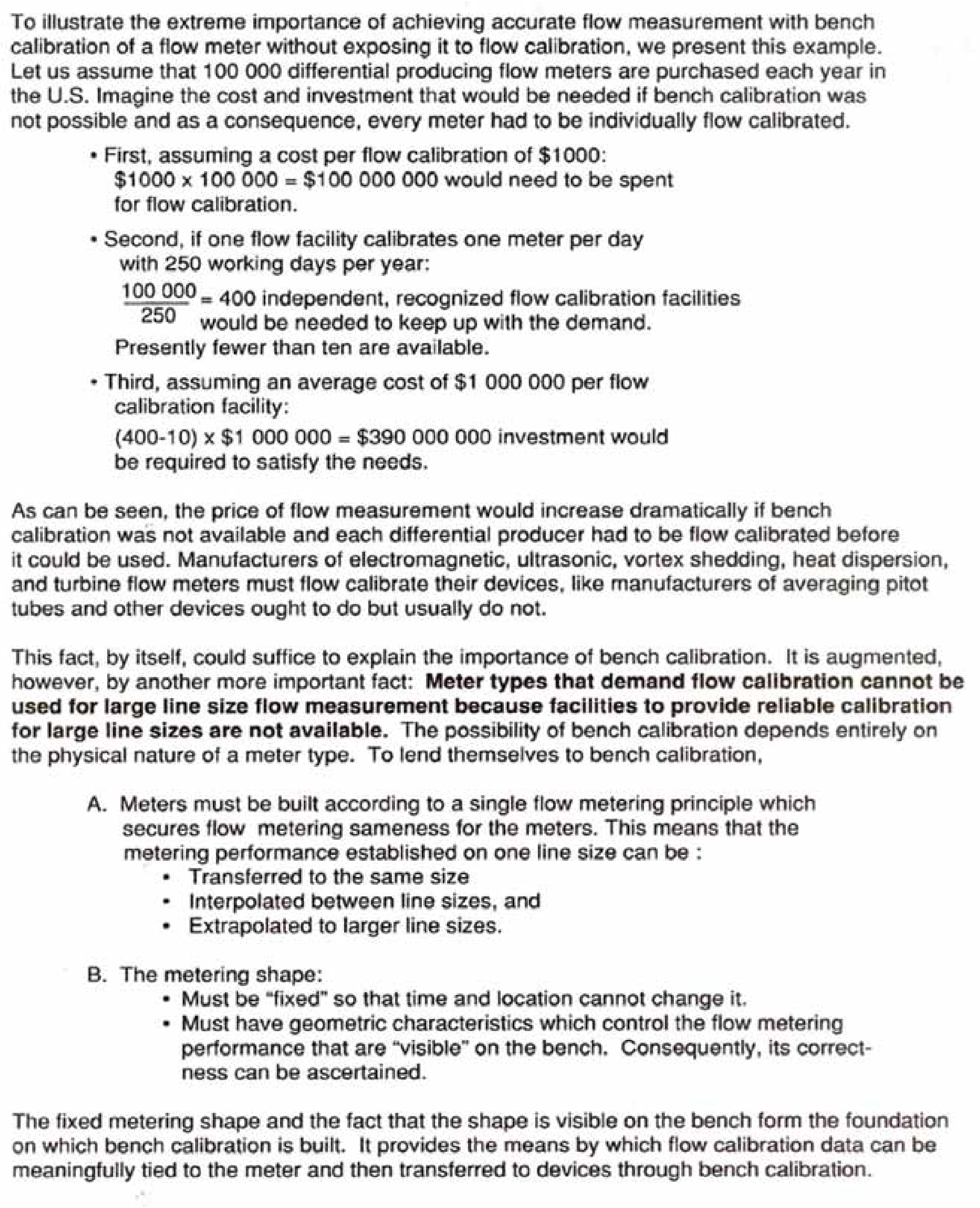
The Importance of Bench Calibration



To illustrate the extreme importance of achieving accurate flow measurement with bench calibration of a flow meter without exposing it to flow calibration, we present this example. Let us assume that 100 000 differential producing flow meters are purchased each year in the U.S. Imagine the cost and investment that would be needed if bench calibration was not possible and as a consequence, every meter had to be individually flow calibrated.

- First, assuming a cost per flow calibration of $1000:
  $1000 x 100 000 = $100 000 000 would need to be spent for flow calibration.
- Second, if one flow facility calibrates one meter per day with 250 working days per year:
  $\frac{100\,000}{250}$ = 400 independent, recognized flow calibration facilities would be needed to keep up with the demand.
  Presently fewer than ten are available.
- Third, assuming an average cost of $1 000 000 per flow calibration facility:
  (400-10) x $1 000 000 = $390 000 000 investment would be required to satisfy the needs.

As can be seen, the price of flow measurement would increase dramatically if bench calibration was not available and each differential producer had to be flow calibrated before it could be used. Manufacturers of electromagnetic, ultrasonic, vortex shedding, heat dispersion, and turbine flow meters must flow calibrate their devices, like manufacturers of averaging pitot tubes and other devices ought to do but usually do not.

This fact, by itself, could suffice to explain the importance of bench calibration. It is augmented, however, by another more important fact: **Meter types that demand flow calibration cannot be used for large line size flow measurement because facilities to provide reliable calibration for large line sizes are not available.** The possibility of bench calibration depends entirely on the physical nature of a meter type. To lend themselves to bench calibration,

A. Meters must be built according to a single flow metering principle which secures flow metering sameness for the meters. This means that the metering performance established on one line size can be :
  - Transferred to the same size
  - Interpolated between line sizes, and
  - Extrapolated to larger line sizes.

B. The metering shape:
  - Must be "fixed" so that time and location cannot change it.
  - Must have geometric characteristics which control the flow metering performance that are "visible" on the bench. Consequently, its correctness can be ascertained.

The fixed metering shape and the fact that the shape is visible on the bench form the foundation on which bench calibration is built. It provides the means by which flow calibration data can be meaningfully tied to the meter and then transferred to devices through bench calibration.

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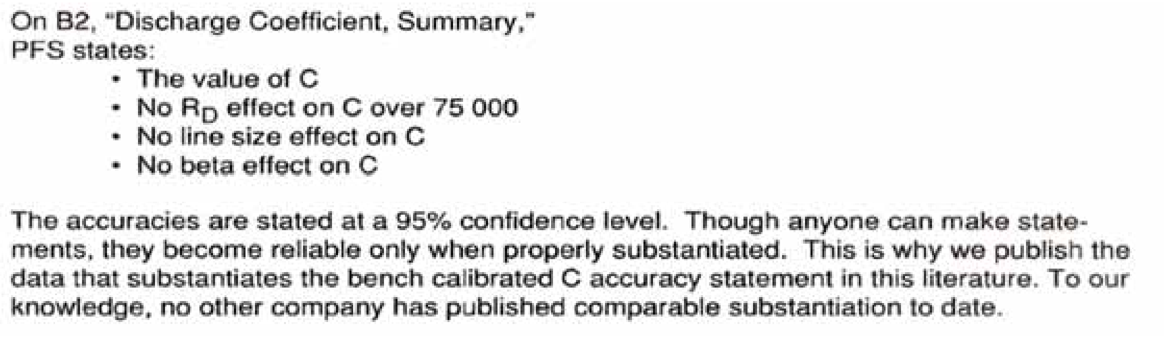
The Reliability of the HVT C

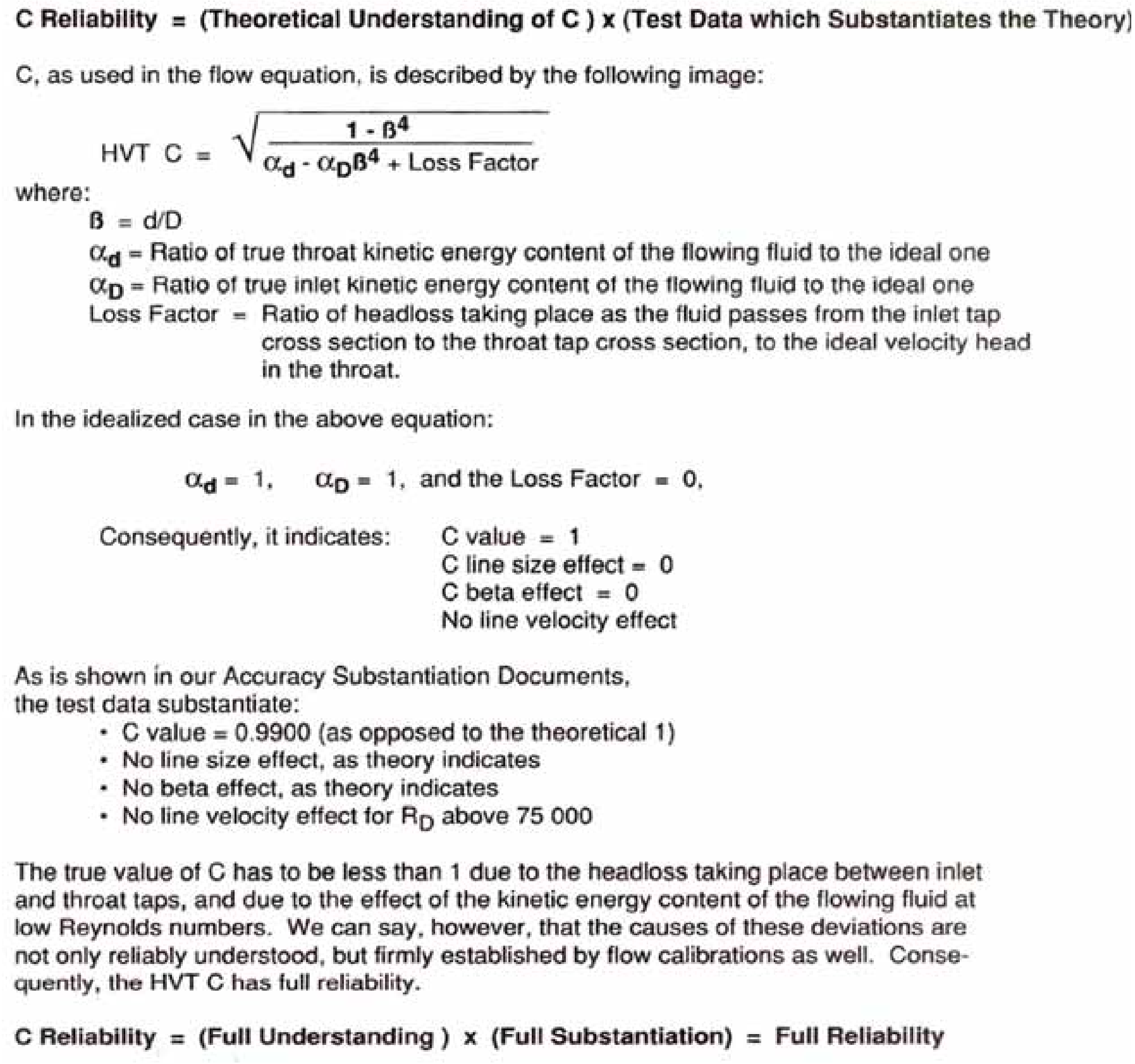


On B2, "Discharge Coefficient, Summary,"
PFS states:

- The value of C
- No R_D effect on C over 75 000
- No line size effect on C
- No beta effect on C

The accuracies are stated at a 95% confidence level. Though anyone can make statements, they become reliable only when properly substantiated. This is why we publish the data that substantiates the bench calibrated C accuracy statement in this literature. To our knowledge, no other company has published comparable substantiation to date.

The Reliability of the Bench Calibrated C



C Reliability = (Theoretical Understanding of C) x (Test Data which Substantiates the Theory)

C, as used in the flow equation, is described by the following image:

$$\text{HVT C} = \sqrt{\frac{1 - \beta^4}{\alpha_d - \alpha_D \beta^4 + \text{Loss Factor}}}$$

where:

$\beta = d/D$

α_d = Ratio of true throat kinetic energy content of the flowing fluid to the ideal one

α_D = Ratio of true inlet kinetic energy content of the flowing fluid to the ideal one

Loss Factor = Ratio of headloss taking place as the fluid passes from the inlet tap cross section to the throat tap cross section, to the ideal velocity head in the throat.

In the idealized case in the above equation:

$$\alpha_d = 1, \quad \alpha_D = 1, \text{ and the Loss Factor } = 0,$$

Consequently, it indicates:
C value = 1
C line size effect = 0
C beta effect = 0
No line velocity effect

As is shown in our Accuracy Substantiation Documents,
the test data substantiate:

- C value = 0.9900 (as opposed to the theoretical 1)
- No line size effect, as theory indicates
- No beta effect, as theory indicates
- No line velocity effect for R_D above 75 000

The true value of C has to be less than 1 due to the headloss taking place between inlet and throat taps, and due to the effect of the kinetic energy content of the flowing fluid at low Reynolds numbers. We can say, however, that the causes of these deviations are not only reliably understood, but firmly established by flow calibrations as well. Consequently, the HVT C has full reliability.

C Reliability = (Full Understanding) x (Full Substantiation) = Full Reliability

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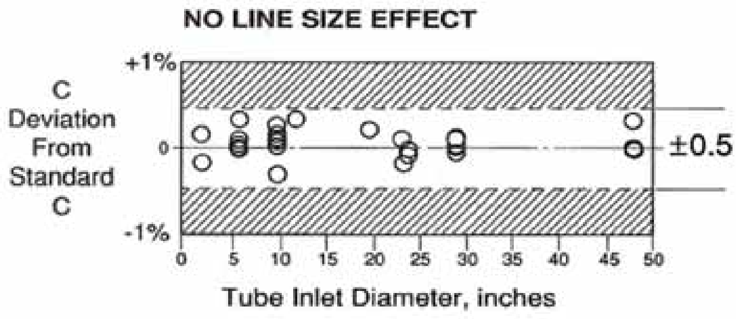
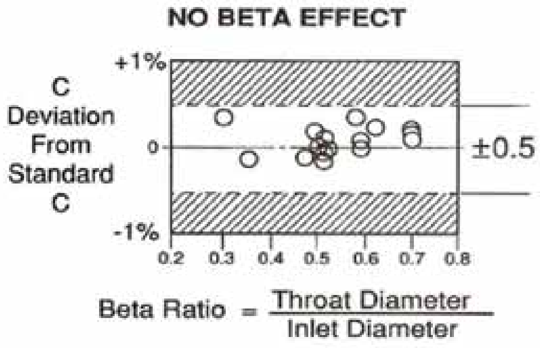
HVT C Substantiation





Substantiation of the Bench Calibrated C Accuracy Statement

PFS states the following about the nature (value, behavior, and accuracy) of C for HVTs:

- Value of C = 0.9900
  For corner inlet tap, C_C = 0.9900 x Tap Factor
- No line size effect
- No beta effect
- No velocity effect if pipe Reynolds number is over 75 000 (R_D behavior)
- C_B accuracy as shown on "Discharge Coefficient Summary" on page B2

Since C statements are not reliable unless substantiated by data from properly executed flow calibrations, we present the following documents:

"SUMMARY OF CALIBRATION DATA"

Document E9 substantiates the fact that the 0.9900 standard coefficient value:

- Is independent of line size as proven by flow calibrations of meters covering 2" to 48" line sizes;
- Is independent of beta ratio as proven by flow calibrations covering 0.31 to 0.75 beta ratio range;
- Has ± 0.41% accuracy, where the accuracy is calculated according to the method described earlier, and consequently covers the effects (if any) of:
  - Line size over a 24:1 line size range
  - Beta ratio through 2.4:1 beta ratio range
  - Inlet tapping effect of static and corner tapping
  - Four physically independent calibrating facilities

It must be emphasized that **this is the only way to substantiate bench calibrated accuracy statements** for any flow meter.

"HVT DISCHARGE COEFFICIENT - PIPE REYNOLDS NUMBER BEHAVIOR"

Document E10 substantiates the PFS statement that the flow rate has no effect on C if R_D is greater than 75 000.

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HVT C Accuracy & Reliability E-9

Summary of Calibration Data

Nominal Inlet Diameter	Beta Ratio	Flow Calibration Facility	Standard HVT Discharge Coefficient	Inlet Tap Factor	Actual Discharge Coefficient	Flow Calibrated Discharge Coefficient	Discharge Coefficient Deviation	n
2.00	0.4822	ARL, Bldg. 2 - 10 000 lb Tank	0.9900	1.0000	0.9900	0.9888	-0.12%	1
2.00	0.5018	ARL, Bldg. 2 - 10 000 lb Tank	0.9900	1.0000	0.9900	0.9919	+0.19%	2
6.00	0.3142	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9935	+0.35%	3
6.00	0.4730	ARL, Bldg. 2 - 10 000 lb Tank	0.9900	0.9884	0.9785	0.9748	-0.38%	4
6.00	0.5999	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9906	+0.06%	5
6.00	0.5999	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9899	-0.01%	6
6.00	0.5999	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	0.9814	0.9716	0.9728	+0.12%	7
6.00	0.5999	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9814	0.9716	0.9720	+0.04%	8
10.00	0.3601	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9868	-0.32%	9
10.00	0.4738	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	0.9907	0.9808	0.9827	+0.19%	10
10.00	0.7059	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9912	+0.12%	11
10.00	0.7060	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9907	+0.07%	12
10.00	0.7060	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9917	+0.17%	13
10.00	0.7507	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	0.9452	0.9357	0.9365	+0.08%	14
10.00	0.7507	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9452	0.9357	0.9362	+0.05%	15
10.00	0.7555	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	0.9432	0.9338	0.9364	+0.28%	16
12.00	0.5875	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9934	+0.34%	17
12.00	0.5875	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9779	0.9681	0.9716	+0.36%	18
18.00	0.4996	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	1.0000	0.9900	0.9865	-0.35%	19
18.00	0.4996	ARL, Bldg. 2 - 50 000 lb Tank	0.9900	0.9916	0.9817	0.9805	-0.12%	20
20.00	0.6307	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9922	+0.22%	21
24.00	0.5240	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9899	-0.01%	22
24.00	0.5240	ARL, Bldg. 2 - Master	0.9900	0.9897	0.9798	0.9790	-0.08%	23
24.00	0.5262	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9923	+0.23%	24
24.00	0.5262	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9874	-0.26%	25
24.00	0.5263	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9909	+0.09%	26
24.00	0.5378	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9832	0.9734	0.9755	+0.22%	27
29.00	0.5184	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9898	-0.02%	28
29.00	0.5184	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9902	0.9803	0.9812	+0.09%	29
29.00	0.5205	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9891	-0.09%	30
29.00	0.5205	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9900	0.9801	0.9812	+0.11%	31
29.00	0.5206	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	1.0000	0.9900	0.9897	-0.03%	32
29.00	0.5206	ARL, Bldg. 1 - 100 000 lb Tank	0.9900	0.9900	0.9801	0.9798	-0.03%	33
36.00	0.5828	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9867	-0.33%	34
36.00	0.5828	ARL, Bldg. 2 - Master	0.9900	0.9836	0.9738	0.9721	-0.17%	35
48.00	0.5271	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9895	-0.05%	36
48.00	0.5271	ARL, Bldg. 2 - Master	0.9900	0.9894	0.9795	0.9778	-0.17%	37
48.00	0.5294	ARL, Bldg. 2 - Master	0.9900	1.0000	0.9900	0.9894	-0.06%	38
48.00	0.5294	ARL, Bldg. 2 - Master	0.9900	0.9893	0.9794	0.9829	+0.36%	39

Reynolds Number Range: 60 000 to 4 300 000

- σ = Standard Deviation $= \pm\sqrt{\frac{\Sigma\Delta C^2}{n-1}} = \pm 0.202\%$ of C
- R = Reproducibility of C for a New Meter $= 2\sigma = \pm 0.404\%$ of C
- P = C Precision $= \pm\frac{t \times \sigma}{\sqrt{n}} = \pm 0.065\%$ of C

  $t = 2.02$ = Student's t for 95% confidence level for 38 (n - 1) degrees of freedom
- A_B = Bench Calibrated C Accuracy $= \pm\sqrt{P^2 + R^2} = \pm 0.41$ % of C

Certified by:

D. Halmi, Engineering

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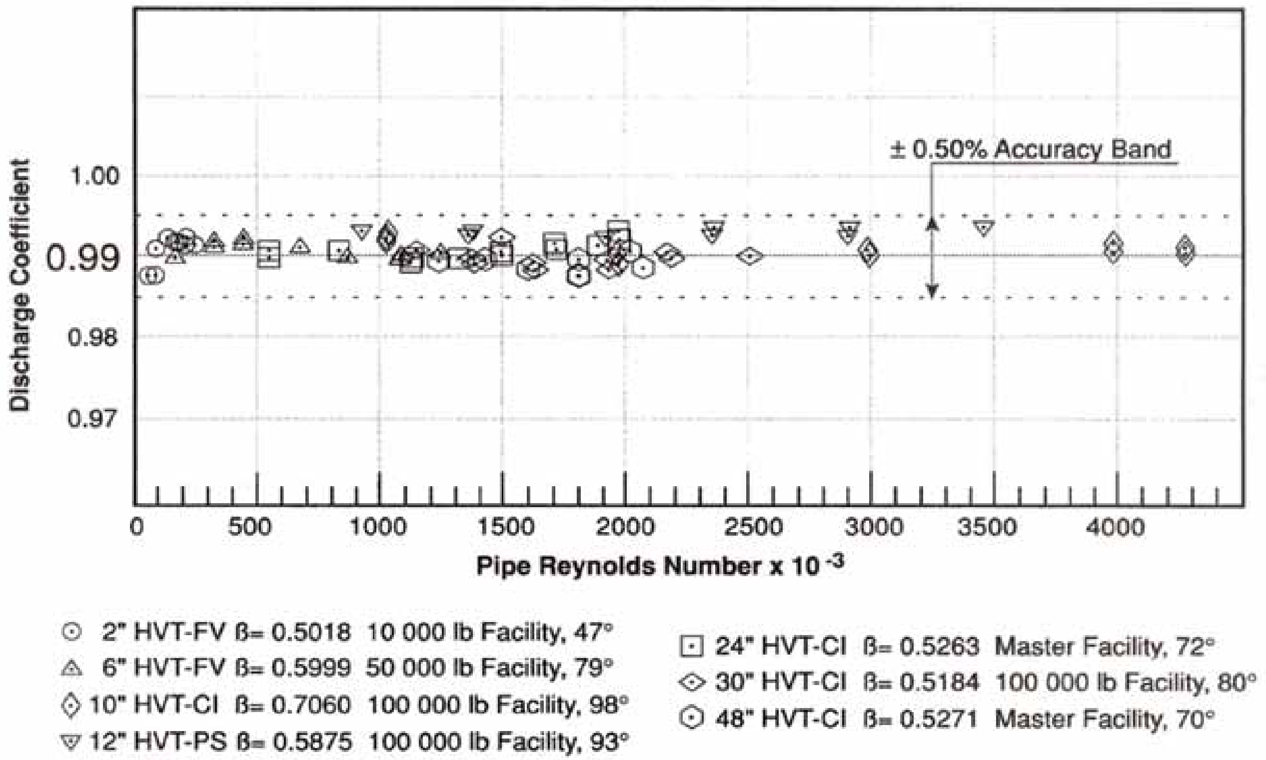
HVT Discharge Coefficient
Pipe Reynolds Number Behavior



Note:
Flow calibrations were performed at Alden Research Laboratory, Inc., Holden, Massachusetts in the flow calibration facilities shown.

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